Multiphysics and multiscale earthqauke modeling

C. Meng (cmeng@mit.edu) B. Hager

ERL, MIT

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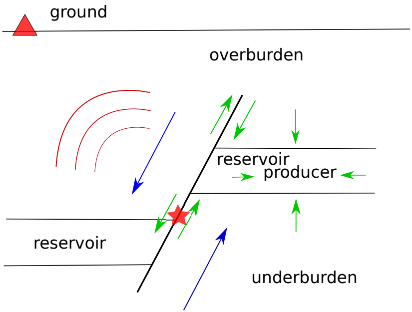
Multiphysics

Poro(visco-)elasticity, (quasi-)static and dynamic processes.

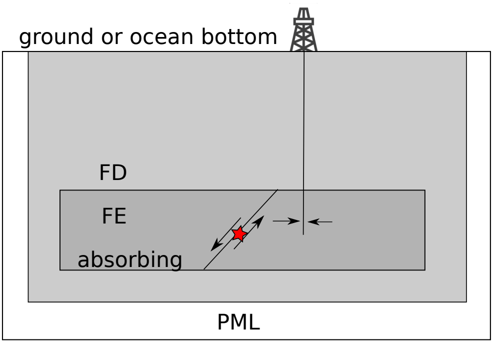


- Static equation $\mathbf{KU} = \mathbf{F}$,
  where $\mathbf{K} = \begin{bmatrix} \mathbf{K}_e & \mathbf{H} \\ -\mathbf{H}^T & \Delta t \mathbf{K}_c + \mathbf{S}_p \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} \mathbf{f} \\ \mathbf{Q} - \Delta t \mathbf{K}_c \mathbf{p} \end{bmatrix}$.
- Dynamic equation $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}_e \mathbf{u} = \mathbf{f}$.

Multi(temporal-)scale

- Implicitly solving the (quasi-)static equation, although expensive, one can take long time step hours, days ...
  $\mathbf{U} = \mathbf{K}^{-1}\mathbf{F}$
- Explicitly solving the dynamic equation, although cheap, one has to take short time step (CFL condition).
  $\mathbf{u}_n = \mathbf{M}^{-1}\left(\Delta t^2(\mathbf{f}_n - \mathbf{K}\mathbf{u}_{n-1}) - \Delta t\mathbf{C}\left(\mathbf{u}_{n-1} - \mathbf{u}_{n-2}\right)\right) + 2\mathbf{u}_{n-1} - \mathbf{u}_{n-2}$

Multi(spatial-)scale



- High order explicit FE methods (SEM, DG) suffer strict CFL condition, and do not solve (quasi-)static equations.
- Linear FE can solve both the static and dynamic equations, but suffers strict CFL condition, i.e. expensive for field scale ground motions.

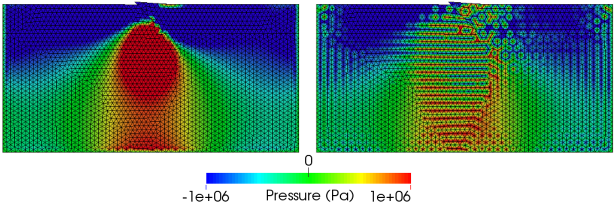
Why a new code?

`https://github.com/Chunfang/defmod-swpc`

	this	Pylith	SPECFEM	Seissol	Comsol
method	FE-FD	FE	SEM	DG	FE
static	✓	✓			✓
dynamic	✓	✓	✓	✓	✓
hybrid	✓				?
poroelastic	✓		?		✓
multiscale	✓				
open	✓	✓	✓	✓	

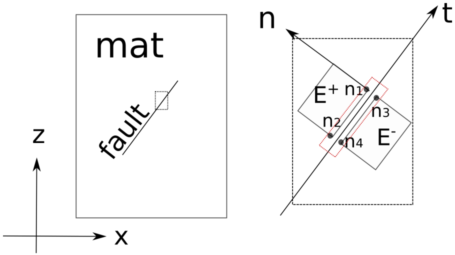
Pore pressure stabilization

Smith and Griffiths (1982) provide the FE formulation for poroelasticity. Bochev and Dohrmann (2006) provide a pore pressure stabilization method.



For Maxwell power law viscoelasticity, the deformation has affect on both the stiffness matrix **K** and the RHS function **F**, Melosh and Raefsky (1980).

Fault constrain equations



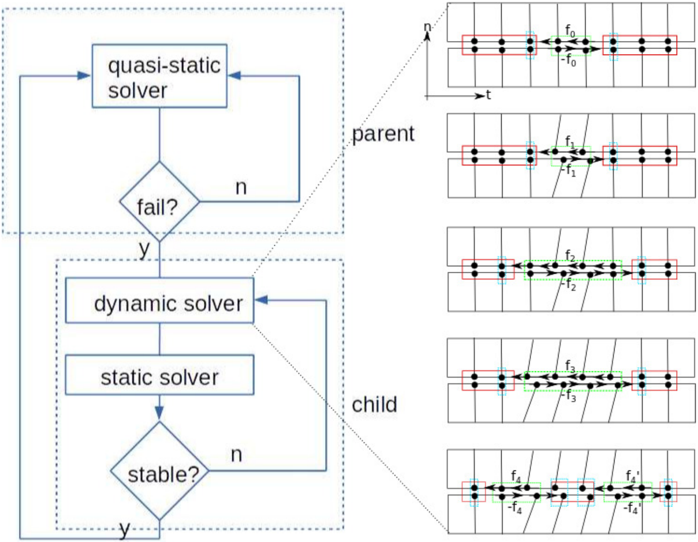
$$\begin{bmatrix} n_x & n_z & 0 & -n_x & -n_z & 0 \\ t_x & t_z & 0 & -t_x & -t_z & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u_x^{(1)} \\ u_z^{(1)} \\ p^{(1)} \\ u_x^{(3)} \\ u_z^{(3)} \\ p^{(3)} \end{bmatrix} = \mathbf{l},$$

$$\begin{bmatrix} \mathbf{K} & \mathbf{G}^T \\ \mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{U}_n \\ \lambda_n \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{f}_n \\ \mathbf{l}_n \end{bmatrix}.$$

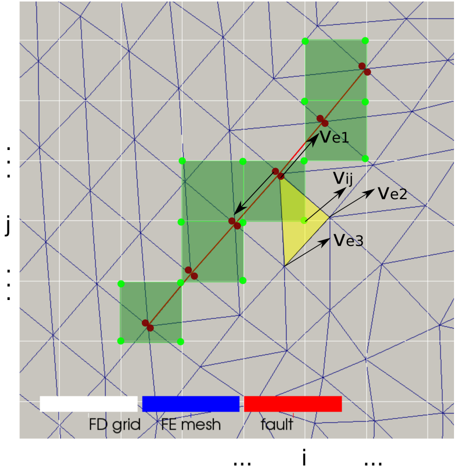
λ is a fault stress proxy. Constraint **l** can be solved for dynamic rupture.

Bartolomeo et al. (2010) provide an explicit routine to model the fault constraint in 2D. Defmod (Meng 2016) has made the method for 3D and general constitutive laws.

Implicit explicit hybrid solver

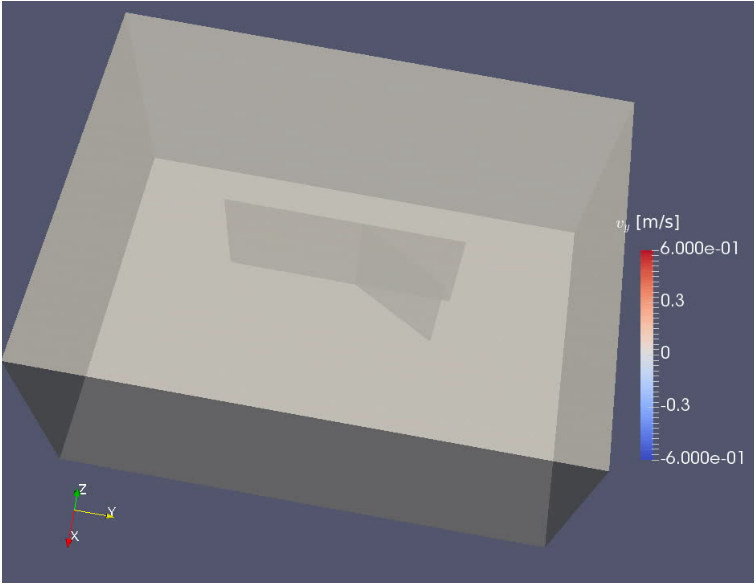


FE-FD direct binding



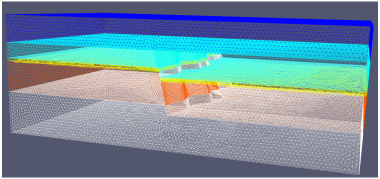
$$\mathbf{v}_{i,j} = \sum_{k=1}^{3} N(\mathbf{x}(i,j),k)\mathbf{v}_{e,k}$$

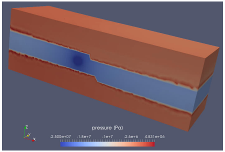
Splay fault rupture

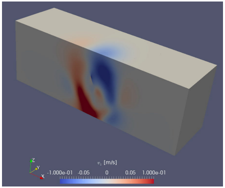


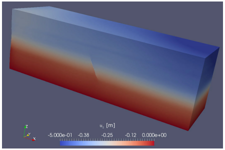
Dipping and curved fault

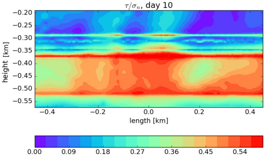
reservoir depletion
induced rupture,
curved fault.

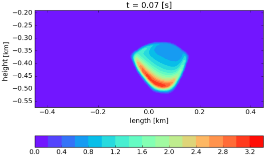


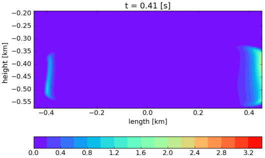


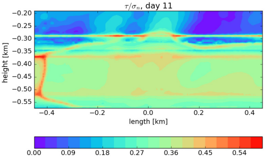








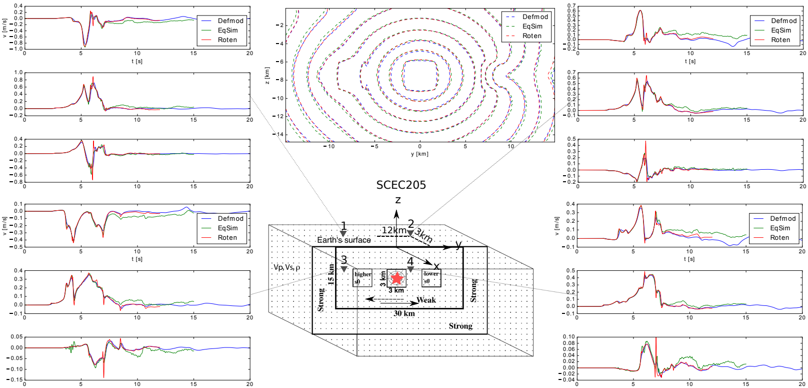




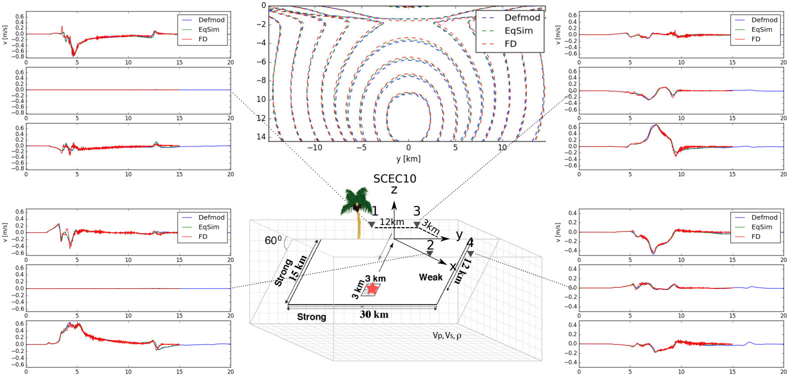
Summary

- This work has resulted a multipysics and multiscale earthquake simulation tool, defmod-swpc.
- Temporally, it covers both the (quasi-)static and dynamic processes.
- Spatially, it covers both the near source/fault motions and far field ground motions.

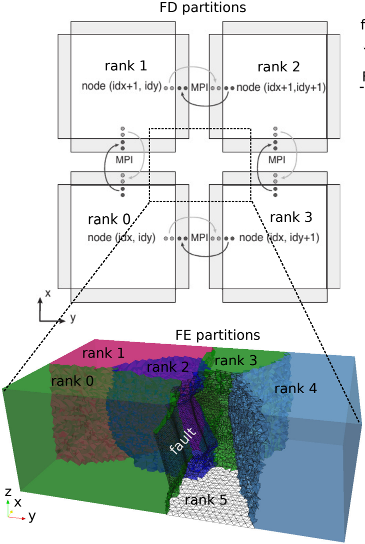
Heterogeneous fault rupture



Dipping fault rupture



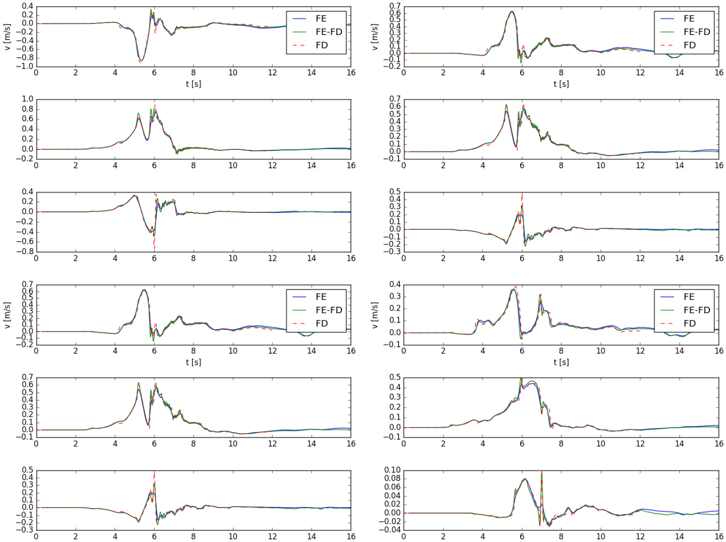
FE-FD parallel connection



fault grid points held by FE/FD ranks

FE \ FD	0	1	2	3
0	1045	0	0	0
1	0	0	0	0
2	251	679	560	45
3	0	458	1022	0
4	0	0	0	0
5	107	45	315	1459

FE-FD verification SCEC 205, pure FD by Cui et al. (2010)



FE-FD verification SCEC 10, pure FD by Andrews (1999)