Quantifying the uncertainty in fault plane solutions inferred from S/P amplitude ratios

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SUMMARY

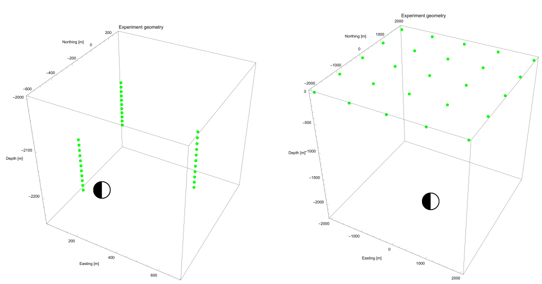
We study the problem of the moment tensor inversion of a double-couple microseismic source from observed S/P amplitude ratios. The emphasis of this work is on uncertainty quantification that includes the effect of the uncertain event location. We use a Bayesian approach to quantify the uncertainty of the fault plane solution. The posterior distribution is effectively calculated by sampling from the posterior distribution of the event location, and performing a moment-tensor inversion using individual samples. The uncertainty in the reconstructed moment tensor depends on the receiver geometry, signal noise, and the true moment tensor. After a suitable transformation of the input data, the problem can be reduced to a classical least-squares estimation problem.

INTRODUCTION

The problems of microseismic event location and moment tensor inversion remains a challenging problem. These problems are particularly important in hydraulic fracture monitoring. Data used for inversion are often incomplete, erroneous, or noisy, and the inversion result therefore may carry significant uncertainty. The location problem in the presence of signal noise and uncertain velocity has received considerable attention (Eisner et al., 2009; Maxwell, 2014; Poliannikov et al., 2014). Here we primarily focus on the problem of moment tensor inversion. The goal is therefore to invert for the moment tensor of a microseismic event and quantify the associated uncertainty. The uncertainty in the reconstructed moment tensor depends on the true moment tensor, acquisition geometry, signal noise, velocity model, attenuation, etc. In this paper, we will focus on the effect of signal noise and location uncertainty on the moment tensor reconstruction. We will construct the fault plane solution using S/P (or P/S) amplitude ratios. Employing S/P ratios, as opposed to direct amplitudes, is more convenient because the ratios do not explicitly depend on the seismic moment, and they appear more stable in the presence of unknown velocity perturbations and/or attenuation. Moment tensor inversion using observed S/P amplitude ratios fitting has been tried with success under deterministic assumptions (Sarkar, 2008; Li et al., 2011a,b; Eyre and van der Baan, 2015). When signal is contaminated by noise, and other model parameters are uncertain, S/P ratios become random variables, i.e., they carry uncertainty. Fitting predicted S/P ratios to observed S/P ratios should be done using a well-suited metric. We use Geary-Hinkley transform to map S/P ratios to nearly-Gaussian quantities. The problem is then optimally solved by using least-squares fitting on these transformed quantities. We illustrate the proposed algorithm using numerical tests with a borehole- and surface-monitoring geometries.

PROBLEM SETUP

Consider a single source, located at $\mathbf{x}_s$ in the subsurface, emitting elastic waves. We assume for simplicity of presentation that the source is double-couple with a strike ϕ, dip δ, and rake λ (Rutledge and Phillips, 2003; Li et al., 2011a,b). An array of receivers located at $\mathbf{x}_{r,j},\ j = 1,\ldots,N_r$, records direct arrivals from the source. We consider two popular monitoring setups shown in Figure 1: a surface array and three borehole arrays (Gu and Toksöz, 2014). The problem is to locate the event and find its moment



(a) (b)

Figure 1: *Experiment geometries with a single source and three borehole arrays or a surface array of receivers.*

tensor with uncertainty quantification for both the location and moment tensor. In this work, we will assume that the uncertainties in the location and the moment tensor are due to signal noise, whereas the velocity models, V_P and V_S, are homogeneous and assumed known. We will seek to estimate the location of the event by constructing a spatial posterior probability distribution, $p(\mathbf{x}_s)$ that describes possible locations of the event given uncertainty in the input parameters of the inversion. We will also construct a fault plane solution with uncertainty by calculating a posterior probability distribution, $p(\phi,\delta,\lambda)$, in the space of strike ϕ, dip δ, and rake λ. The moment tensor uncertainty will include the direct effect of the signal noise on the recorded amplitudes, as well as the effect of the uncertainty in the event location, assumed to be derived in a previous analysis. In order to construct these probability distribution we will use the classical Bayesian inversion approach.

BAYESIAN INVERSION

Bayes' rule provides the posterior distribution $p(m \mid d)$ of the unknown model m given observed data d:

$$p(m \mid d) = \frac{p(d \mid m)\, p(m)}{p(d)} \propto p(d \mid m), \tag{1}$$

where $p(d \mid m)$ is a likelihood function (forward or measurement model), $p(m)$ is a prior distribution of the model parameters, and $p(d)$ is the normalizing constant, which is an unconditional probability of data. The proportionality sign in Equation 1 assumes an uninformed (flat) prior.

In this problem the unknown model parameters are the location $\mathbf{x}_\mathrm{s}$, and the angles ϕ, δ, and λ. The continuous 3C seismic data have the form:

$$u_{nj}(t) = \sum_{p,q} M_{pq} \star G_{np,q}(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s}, t), \tag{2}$$

where $M \equiv M(\phi, \delta, \lambda)$ is the moment tensor, and G is the Green's function.

We assume that the continuous traces are processed and a set of discrete waveform attributes is extracted. Specifically, we denote as $\hat{T}_{P,j}$, $\hat{T}_{S,j}$ the estimated arrival times of P and S waves at the jth receiver, and $\hat{A}_{P,j}, \hat{A}_{S,j}$ — the observed amplitudes of P and S wave at the jth receiver. We will assume that all errors, including errors in the estimated arrival times and amplitudes, are mutually uncorrelated.

LOCATING THE EVENT

To locate the event, we follow Poliannikov et al. (2014). We assume that arrival times, $\hat{\mathbf{T}} = \left\{\hat{T}_{\alpha,j} \mid \alpha \in \{P,S\},\ j \in \{1,\ldots,N_\mathrm{r}\}\right\}$, can be modeled as follows:

$$\hat{T}_{\alpha,j} = \mathring{T} + T\left(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s} \mid V_\alpha\right) + \sigma_{\alpha,j} n_{\alpha,j}, \tag{3}$$

where $\mathring{T}$ is the unknown source origin time, $T\left(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s} \mid V_\alpha\right)$ is the predicted travel time from $\mathbf{x}_\mathrm{s}$ to $\mathbf{x}_{\mathrm{r},j}$ in the velocity V_α, $\{n_{\alpha,j}\}$ are independent random variables with $N(0,1)$ distribution, and $\sigma_{\alpha,j}$ are noise variances, assumed known.

Then the event location posterior has the form:

$$p(\mathbf{x}_\mathrm{s} \mid \hat{\mathbf{T}}) \propto \exp\left(\frac{B^2}{2A} + C\right), \tag{4}$$

where

$$A = \sum_{\alpha,j} \frac{1}{\sigma^2_{\alpha,j}}, \tag{5}$$

$$B = \sum_{\alpha,j} \frac{\hat{T}_{\alpha,j} - T\left(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s} \mid V_\alpha\right)}{\sigma^2_{\alpha,j}}, \tag{6}$$

$$C = -\frac{1}{2}\sum_{\alpha,j} \frac{\left(\hat{T}_{\alpha,j} - T\left(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s} \mid V_\alpha\right)\right)^2}{\sigma^2_{\alpha,j}}. \tag{7}$$

Equation 4 accounts for the unknown origin time. In this form, it also assumes an uninformed prior on $\mathbf{x}_\mathrm{s}$. Readers are referred to Poliannikov et al. (2014) for formulas with the prior included and more information on uncertainty quantification for event location. Equation 4 provides the full 3D posterior, as well as 2D and 1D marginal distributions, as shown in Figures 2 and 3.

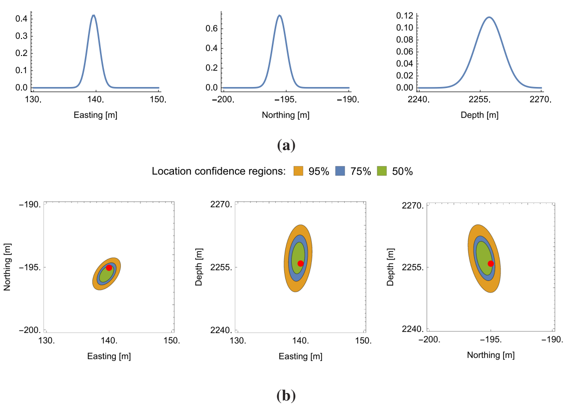


Figure 2: *Location uncertainty estimate from borehole receivers*

ESTIMATING THE FAULT PLANE SOLUTION

We now consider the problem of estimating the moment tensor of this event by taking the location uncertainty calculated above into account. We will use the same Bayesian approach to solve this problem. For any location $\mathbf{x}_\mathrm{s}$ and any fault plane solution $M \equiv (\phi, \delta, \alpha)$, we can model the amplitudes of the P and S arrival, $A_\alpha(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s})$, using Equation 2. The observed amplitudes are noisy versions of the predicted amplitudes

$$\hat{A}_{\alpha,j} = A_\alpha(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_\mathrm{s}) + \zeta_{\alpha,j}\, \nu_{\alpha,j}. \tag{8}$$

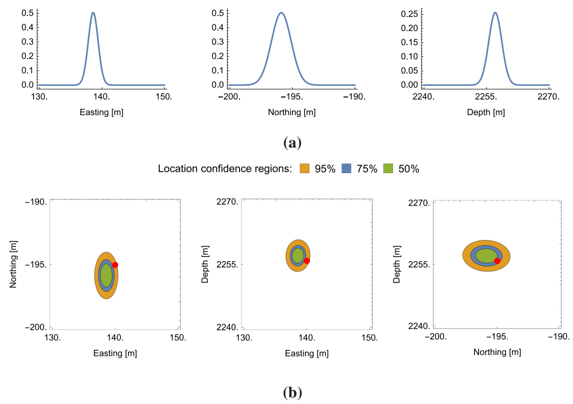


Figure 3: *Location uncertainty estimate from surface receivers*

Because the angles (ϕ, δ, λ) define a double-couple moment tensor up to a multiplicative seismic moment, we remove this ambiguity, as is customary, by considering amplitude ratios $\{\hat{A}_{S,j}/\hat{A}_{P,j}\}$ (or $\{\hat{A}_{P,j}/\hat{A}_{S,j}\}$) estimated at each receiver j These ratios contain information about the moment tensor. In particular, in the homogeneous case, we will have

$$\frac{A_S(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_{\mathrm{s}})}{A_P(\mathbf{x}_{\mathrm{r},j}, \mathbf{x}_{\mathrm{s}})} = \left(\frac{V_P}{V_S}\right)^3 \frac{R_S(\psi_j, \theta_j \mid M, \mathbf{x}_{\mathrm{s}})}{R_P(\psi_j, \theta_j \mid M, \mathbf{x}_{\mathrm{s}})}, \qquad (9)$$

where ψ_j, θ_j are the azimuth and elevation of the jth receiver relative to an assumed source location $\mathbf{x}_{\mathrm{s}}$, and R_P and R_S are radiation patterns of P and S wave corresponding to the fault plane solution M. When the signals are perturbed by additive noise, the observed amplitudes can be modeled as Gaussian perturbations of the predicted amplitudes. Note that the *ratio* of the two Gaussian-distributed noisy amplitudes is not Gaussian. This is not an impediment for Bayesian inversion. However, it is often convenient to work with Gaussian data because the estimation problem will have familiar solutions. In what follows, we convert our problem to a Gaussian one without any loss of generality. Denote the amplitude ratios as $\hat{\mathbf{r}}_A = \{\hat{A}_{S,j}/\hat{A}_{P,j} \mid j \in \{1, \ldots, N_{\mathrm{r}}\}\}$. The general form for the posterior of the moment tensor is obtained by

$$\begin{aligned} p(M \mid \hat{\mathbf{T}}, \hat{\mathbf{r}}_A) &= \int p(M, \mathbf{x}_{\mathrm{s}} \mid \hat{\mathbf{T}}, \hat{\mathbf{r}}_A)\, d\mathbf{x}_{\mathrm{s}} \\ &= \int p(M \mid \mathbf{x}_{\mathrm{s}}, \hat{\mathbf{T}}, \hat{\mathbf{r}}_A)\, p(\mathbf{x}_{\mathrm{s}} \mid \hat{\mathbf{T}}, \hat{\mathbf{r}}_A)\, d\mathbf{x}_{\mathrm{s}} \\ &\approx \int p(M \mid \mathbf{x}_{\mathrm{s}}, \hat{\mathbf{r}}_A)\, p(\mathbf{x}_{\mathrm{s}} \mid \hat{\mathbf{T}})\, d\mathbf{x}_{\mathrm{s}} \\ &= \mathbb{E}_{\mathbf{x}_{\mathrm{s}}}\left[p(M \mid \mathbf{x}_{\mathrm{s}}, \hat{\mathbf{r}}_A)\right] \end{aligned} \qquad (10)$$

The $\approx$ sign is to underscore an additional assumption we make that amplitudes do not have much additional information about the event location beyond what is already provided by the travel times.

We have developed a numerical algorithm for evaluating the posterior of M based on approximating the expectation ending Equation 10 with a sample mean. That is, we sample possible event locations, $\{\mathbf{x}_{\mathrm{s},i}\}_{i=1}^{N_{\mathrm{s}}}$, from the posterior distribution $p(\mathbf{x}_{\mathrm{s}} \mid \hat{\mathbf{T}})$. For each sample location, we can perform a separate inversion; then average the results:

$$p(M \mid \hat{\mathbf{T}}, \hat{\mathbf{r}}_A) \approx \frac{1}{N_{\mathrm{s}}} \sum_{i=1}^{N_{\mathrm{s}}} p(M \mid \mathbf{x}_{\mathrm{s},i}, \hat{\mathbf{r}}_A). \qquad (11)$$

Geary-Hinkley transform of amplitude ratios

The S/P amplitude ratio is the ratio of two non-zero-mean Gaussian random variables. While we could work with such ratios directly, it is more convenient to transform them into Gaussian random variables with a suitable transform. Note that we do not assume that the ratios have Gaussian distributions. Instead, we apply a transform that ensures that the transformed quantities have a Gaussian distribution. Specifically, suppose that $(\xi_1, \xi_2) \sim N((\mu_1, \mu_2), \Sigma)$ are jointly Gaussian with known means and the covariance matrix. Let $r = \xi_2/\xi_1$ be their ratio. Define

$$\rho = \mathscr{P}(r) = \frac{\mu_2 r - \mu_1}{\sqrt{\Sigma_{22} r^2 - 2\Sigma_{12} r + \Sigma_{11}}}. \qquad (12)$$

It has been shown that ρ is approximately $N(0,1)$, provided ξ_1 is away from zero with a very large probability. Equation 12 is called Geary-Hinkley transform We illustrate numerically how the transform works in Figure 4.

Maximum-likelihood and posterior estimates

We compute maximum-likelihood and posterior estimators of the moment tensor using least squares in the $\mathscr{P}$-domain. The ML estimate is obtained by minimizing the least-squares error between the observed transformed ratios, $\mathscr{P}(\hat{\mathbf{r}}_A)$, and the predicted transformed ratios, $\mathscr{P}(\mathbf{r}_A)$, where $\mathbf{r}_A$ is obtained from Equation 9. The results are shown in Figure 5. The estimators visually appear very good but they do not provide any quantitative information about possible uncertainty. In order to show the uncertainty in the reconstructed fault plane solutions, we calculate the Bayesian posteriors and show them in Figure 6. We observe that the surface network appears to give better results, which is to expected because much of the energy produced by a dip-slip event radiates vertically. The posterior in our examples also does not appear to be Gaussian. *Ad hoc* estimates of the possible range of uncertainty in the absence of a careful calculation may therefore be wholly inadequate. Failure to fully capture

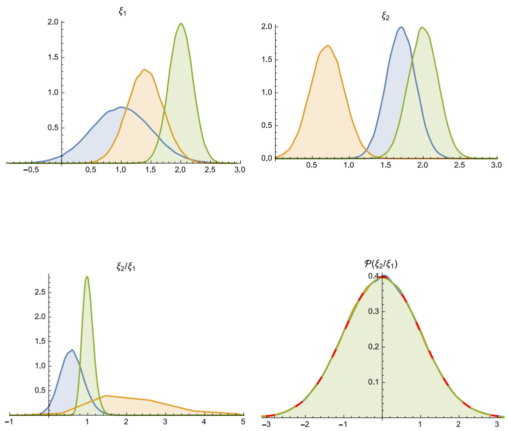
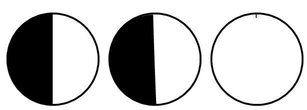
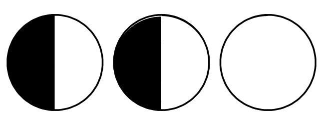


Figure 4: *The ratio, ξ_2/ξ_1, of two Gaussian random variables, ξ_1 and ξ_2, is not Gaussian. It becomes nearly Gaussian after a Geary-Hinckley transform. Different choices of distributions are coded by color.*

this uncertainty may lead to the propagation of errors to next possible stages of analysis, e.g., estimation of the stress state in the reservoir.



(a)

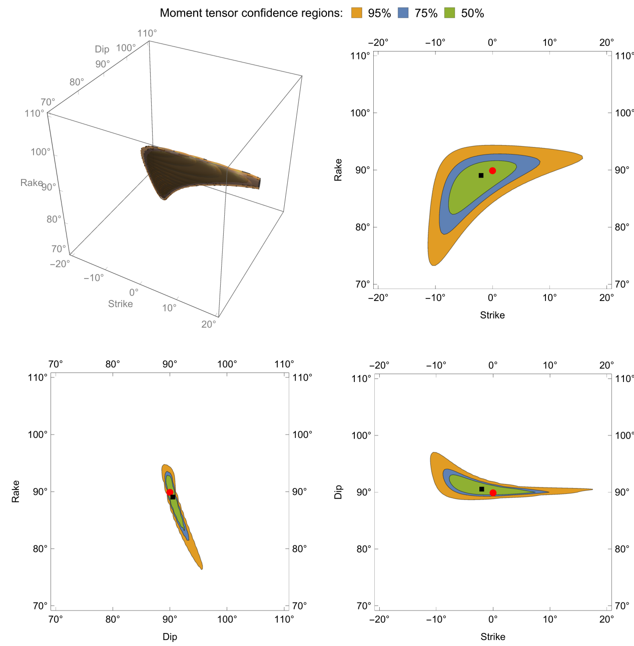


(b)

Figure 5: *True moment tensor (left), maximum likelihood estimators obtained from (a) the boreholes or (b) the surface (middle), and the difference between the true and estimated moment tensors (right).*

CONCLUSIONS

We presented a Bayesian approach to the problem of moment tensor inversion in the presence of signal noise and event location uncertainty. The velocity model was assumed known but the approach can be extended to an uncertain velocity model. The posterior distribution of the angles in the fault plane solution depends on a true moment tensor, geometry of the receiver network, noise strength, and other factors. Results of this type of analysis can be useful in further applications such as stress estimation or acquisition design.



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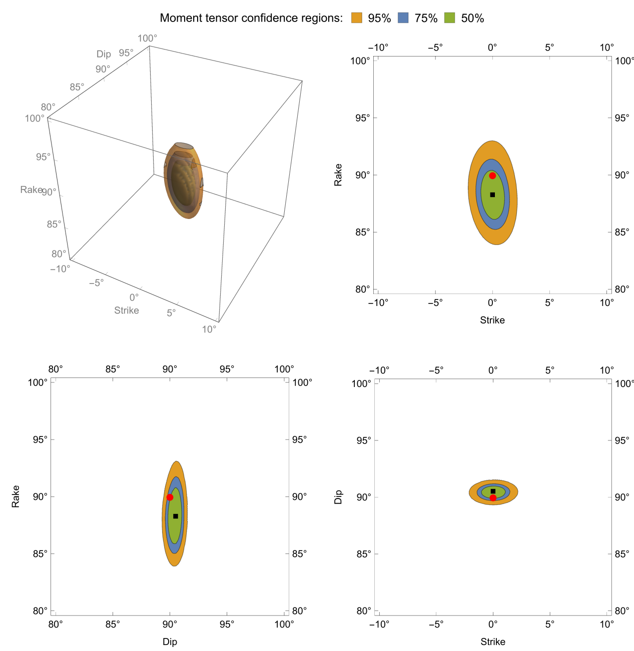

(b)

Figure 6: *Moment tensor confidence regions obtained from data recorded (a) in the borehole and (b) at the surface*

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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